

# CLASSICAL FIELDS IN HEAVY-ION COLLISIONS

**Vladimir Skokov**



Brain workshop

- Raju Venugopalan this morning: Glasma and thermalization

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- Electromagnetic fields acting on in- and out-of-equilibrium plasma

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- Electromagnetic fields acting on in- and out-of-equilibrium plasma
- Color fields in-equilibrium: non-trivial holonomy  $A_0$
- Chiral condensate and manifestation of its dynamics: chiral crossover, chiral critical point, quarkyonic phase

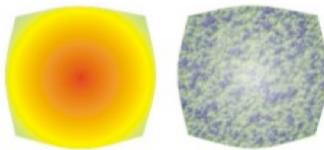
# CLASSICAL FIELD IN HEAVY-ION COLLISIONS

Non-equilibrium



Color glass condensate

Equilibrium



Glasma

$E^a$  &  $B^a$

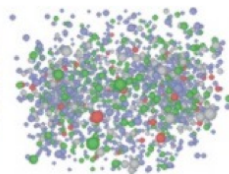
High EM fields

$E$  &  $B^a$

Classical color  $A_0$

(Inhomogeneous) chiral condensate

Freeze-out

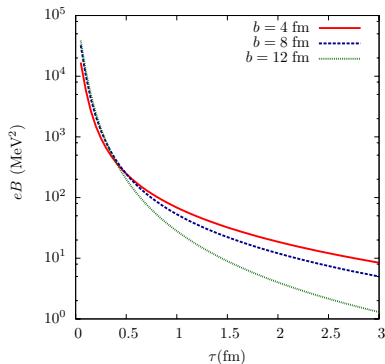


Almost on every stage of heavy-ion collisions, classical fields play significant role.

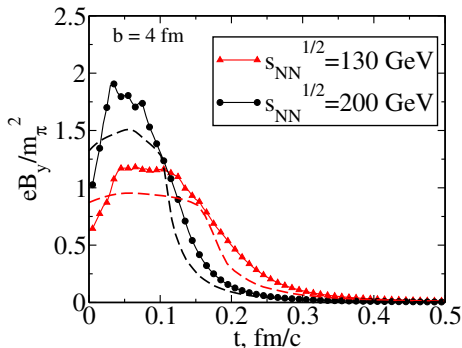
# (ELECTRO)-MAGNETIC FIELD IN HIC

HICs create not only strong color  
but also extremely strong (electro)-magnetic fields.

**Strong**  $\equiv$  of the hadronic scales,  $eB \sim m_\pi^2$ .



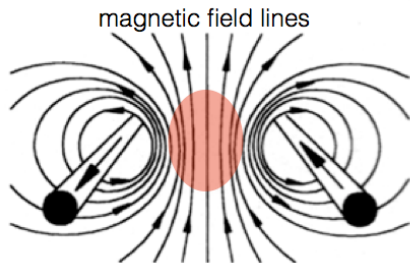
D. Kharzeev, L. McLerran and H. Warringa  
Nucl.Phys. A803 (2008) 227-253



V. S., Y. Illarionov and V. Toneev  
arXiv:0907.1396

# ORIGIN OF (ELECTRO)-MAGNETIC FIELD IN HIC

In the first approximation, colliding nuclei are two positive charges moving with ultra relativistic velocities in opposite directions.

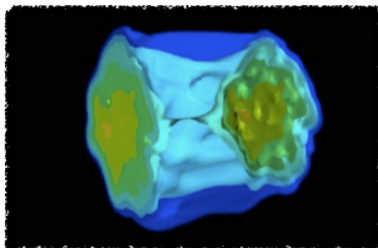


Two currents in opposite direction.  
Magnetic fields of the two sources add up,  
while electric fields nearly cancel each other.

Out-of-plane direction of magnetic field:

$$\langle eB_y \rangle \sim m_\pi^2, \langle eB_x \rangle \sim \langle eB_z \rangle \sim 0$$

$$\langle eE_x \rangle \sim \langle eE_y \rangle \sim \langle eE_z \rangle \sim 0$$



Charge distribution in nuclei is not uniform.

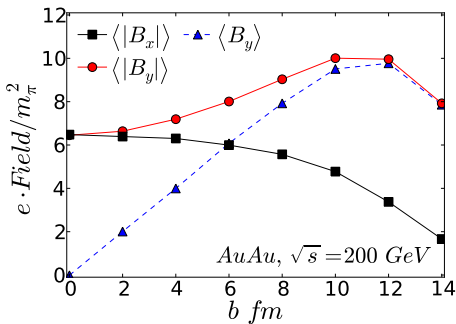
Lumpy distribution of electric charge in  
colliding nuclei results in nonzero randomly  
oriented magnetic field even in central  
collisions.



# FLUCTUATIONS OF (ELECTRO)-MAGNETIC FIELD

A. Bzdak and V.S.

Phys.Lett. B710 (2012) 171



- Fluctuations of participant and reaction planes  
( J. Błoczyński et al, Phys.Lett. B718 (2013) 1529)
- Non-zero electric field can induce separation of charges;  
 $\mathbf{j} = \sigma \mathbf{E}$   
measurements of electric conductivity  
(talk by Y. Hirono)
- Lifetime of (electro)-magnetic field?!

External magnetic field might live longer because of conductivity effects:

$$\mathbf{j} = \sigma_{\text{Ohm}} \cdot \mathbf{E} + \sigma_{\chi} \cdot \mathbf{B}$$

where  $\sigma_{\text{Ohm}}$  is electric conductivity and  $\sigma_{\chi}$  is chiral-magnetic conductivity  
(D. Kharzeev et al).

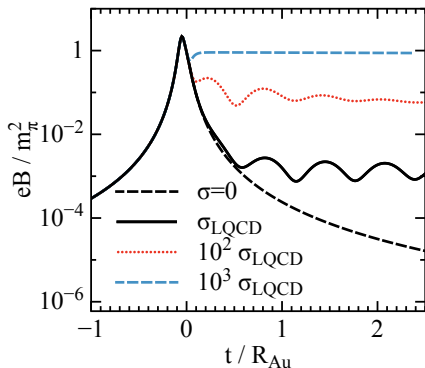
Electric conductivity:

quenched lattice QCD calculations  $\sigma_{\text{Ohm}}^{\text{LQCD}} = (5.8 \pm 2.9) \frac{T}{T_c} \text{ MeV}$ .

Chiral magnetic conductivity:

$$\sigma_{\chi} = \left( \frac{e^2}{2\pi^2} N_c \sum_f q_f^2 \right) \mu_5, \text{ let } \mu_5 \sim 1 \text{ GeV, then } \sigma_{\chi} \sim 15 \text{ MeV}.$$

# LIFETIME OF MAGNETIC FIELD



S. Lin, L. McLerran and V.S.

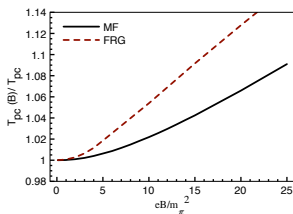
- Even for  $\sigma_{Ohm} = 10^2 \sigma_{Ohm}^{LQCD}$  magnetic field falls quite fast.
- $\sigma_{Ohm} \sim \sigma_{Ohm}^{LQCD}$  is an optimistic estimate, since in early stage there are no charge carriers
- Magnetic field acts on very early (non-equilibrium) nuclear matter

# OBSERVABLE EFFECTS: QCD PHASE DIAGRAM

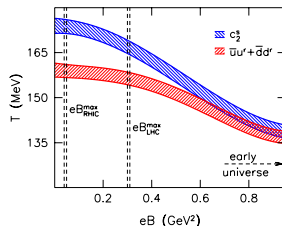
## Modification of QCD phase diagram in high magnetic field.

*Probably* (unless system is in local equilibrium at times less than 0.1 fm/c), not relevant for HIC phenomenology owing to short lifetime of magnetic field.

Nonetheless, as we learned from talk by Y. Hidaka, QCD calculations with magnetic field showed that existing low-energy effective models does not capture essential chiral and deconfinement properties of QCD.



V.S. Phys.Rev. D85 (2012) 034026



G. S. Bali et al JHEP02 (2012) 044

FRG: functional renormalization group will be discussed in talk by Y. Tanizaki

- **Chiral Magnetic Effect and Chiral Magnetic Wave:  $j \sim \sigma_\chi B$ .**
- **Photon production in strong magnetic field**

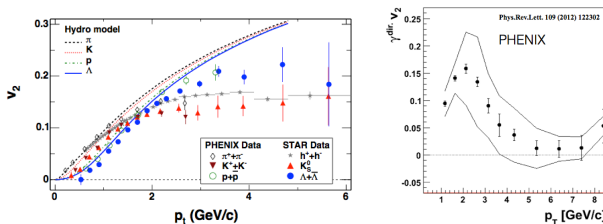
Conformal anomaly: G. Bazar, D. Kharzeev and V.S. Phys.Rev.Lett. 109 (2012) 202303

Chiral anomaly: K. Fukushima and K. Mameda Phys.Rev. D86 (2012) 071501

Ho-Ung Yee, arXiv:1303.3571

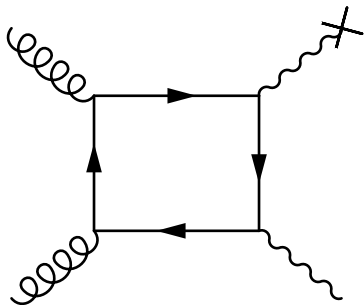
Photon azimuthal anisotropy (quantified by  $v_2$ ) is approximately the same as for charge particles. In contradiction to expectations: photon production  $\sim T^4$ , however, momentum azimuthal anisotropy is small.

Hydrodynamic calculations underestimate photon  $v_2$  by factor of 5.



# PHOTON PRODUCTION

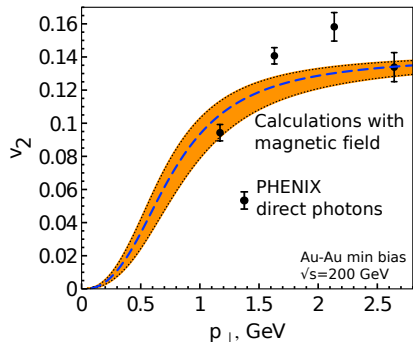
eB



Usually photon production from two gluons is suppressed by  $\alpha_{EM}$  compared to quark-antiquark annihilation or Compton scattering



If magnetic field of hadronic scales  $eB \sim m_\pi^2$ , two gluons produce one photon with rate similar to quark-antiquark annihilation and Compton scattering.



Experimental data was underpredicted by a factor of **5** in hydro calculations.

- Resummed contribution from gluons in color singlet state.
- In vacuum, dilaton is an effective description of Yang-Mills theory.  
Assumption: single scalar meson saturates conformal anomaly.

Dilatation current acting color singlet states

$$\langle 0 | D^\mu | \sigma \rangle = i q^\mu f_\sigma; \quad \langle 0 | \partial_\mu D^\mu | \sigma \rangle = m_\sigma^2 f_\sigma$$

From decay of lowest glueball state to two photons decay to two photons coupling  $g_{\sigma\gamma\gamma}$  of following effective Lagrangian is obtained

$$\mathcal{L}_{\sigma\gamma\gamma} = g_{\sigma\gamma\gamma} \sigma F_{\mu\nu} F^{\mu\nu} \text{ with}$$

$$\sigma \sim \theta_\mu^\mu = \partial^\mu D_\mu = \theta_\mu^\mu = \frac{\beta(g)}{2g} G^{\mu\nu} G_{\mu\nu} + \sum_q m_q (1 + \gamma_m(g)) \bar{q} q,$$

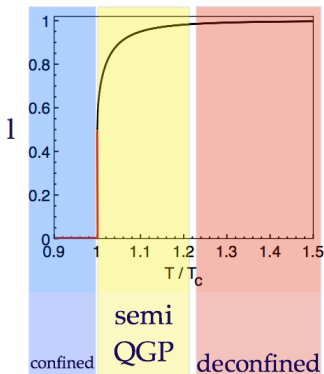
- Photon production rate

$$q_0 \frac{d^3\Gamma}{dq^3} \propto \left[ (B_y^2 - B_x^2) q_x^2 + B x^2 q_\perp^2 \right] \rho_\theta(q_0),$$

where  $\rho_\theta(q_0)$  is spectral function for trace of energy momentum tensor, in hydrodynamical limit  $\rho_\theta(q_0) \approx \frac{9}{\pi} q_0 \zeta$

# NON-TRIVIAL HOLONOMY $A_0$ AT FINITE $T$

- In equilibrium, color background field  $A_0$  is expected
- Characterized by Polyakov loop  $L = \text{Tr } \mathcal{P} \exp \left( ig \int_0^{1/T} A_0 d\tau \right)$ , which also play a role of order parameter for deconfinement in Yang Mills theory



R. Pisarski

- Effective theory to study properties of YM sector
- High  $T$ : effective potential obtained perturbatively

$$V^{\text{pert}} = \frac{2\pi^2}{3} T^4 \left( -\frac{4}{15} (N_c^2 - 1) + \sum_{ij=1}^{N_c} q_{ij}^2 (1 - q_{ij})^2 \right),$$

$$q_{ij} = |q_i - q_j|$$

$$\text{Ansatz } A_0^{ij} = \frac{2\pi T}{g} q_i \delta^{ij}, \quad \sum_i q_i = 0$$

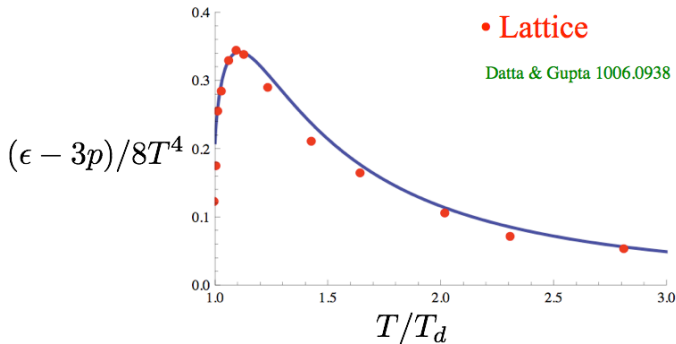
(Gross-Pisarski-Yaffe/Weiss potential)

- Non-trivial solution for  $q_i$  demands non-perturbative potential which is assumed to be
- $$V^{\text{non-pert}} = T^2 T_d^2 \left[ C_1 \sum_{ij=1}^{N_c} q_{ij}^2 (1 - q_{ij})^2 + C_2 \sum_{ij=1}^{N_c} q_{ij} (1 - q_{ij}) + C_3 \right]$$

Further discussion in talk by K. Kashiwa



# COMPARISON TO LATTICE



A. Dumitru et al Phys.Rev. D86 (2012) 105017

Good description of YM thermodynamics, interface tension for SU(2) and SU(3).

- Potential in explicit form

$$V/(N_c^2 - 1) \equiv \widetilde{V}_{\text{eff}}(q) = -d_1(T) \widetilde{V}_1(q) + d_2(T) \widetilde{V}_2(q) ,$$

$$\widetilde{V}_n(q) = 1/(N_c^2 - 1) \sum_{i,j=1}^{N_c} |q_i - q_j|^n (1 - |q_i - q_j|)^n$$

- It is useful to introduce eigenvalue density

$$\rho(q) = \frac{1}{N_c} \sum_i \delta(q - q_i) \rightarrow \int_0^1 dx \delta(q - q(x)) = dx/dq$$

Then

$$1/N_c \sum_i f(q_i) \rightarrow \int_0^1 dx f(q_x) = \int dq \frac{dx}{dq} f(q) = \int dq \rho(q) f(q)$$

- Terms in the potential

$$V_n(q) = \int dq \int dq' \rho(q) \rho(q') |q - q'|^n (1 - |q - q'|)^n .$$

non-local field theory

# SOLUTION IN LARGE $N_c$ LIMIT

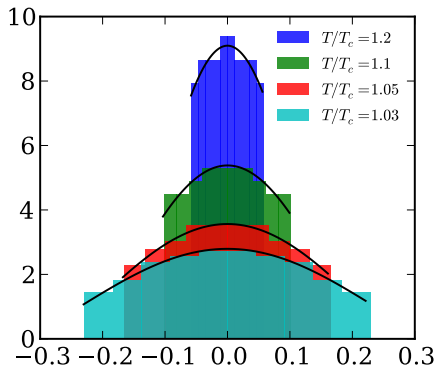
- Solution for eigenvalue density  
( $d = 12d_2/d_1$ )

$$\rho(q) = 1 + b \cos(dq) \quad , \quad q : -q_0 \rightarrow q_0$$

- $q_0$  and  $b$  are given by

$$\cot(dq_0) = \frac{d}{3} \left( \frac{1}{2} - q_0 \right) - \frac{1}{d(1/2 - q_0)} \quad ,$$

$$b^2 = \frac{d^4}{9} \left( \frac{1}{2} - q_0 \right)^4 + \frac{d^2}{3} \left( \frac{1}{2} - q_0 \right)^2 + 1$$



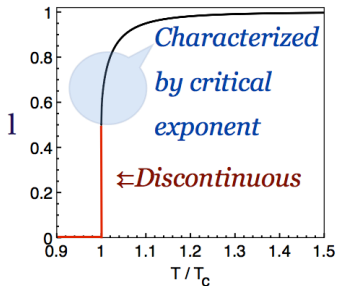
R. Pisarski and V.S.  
Phys.Rev. D86 (2012) 081701

# GROSS-WITTEN-WADIA PHASE TRANSITION

What is order of deconfinement phase transition in large  $N_c$  limit?

Effective matrix model: critical first order phase transition (PT): aspects of both first order and second order PTs.

- Discontinuity as in first order PT. Approaching  $T_d$  Polyakov loop jumps from 0.5 to 0.
- Divergence of order parameter susceptibility as in second order PT.



Critical exponents obtained analytically:

$$l \sim (T/T_d - 1)^\beta, \quad \beta = 2/5$$

$$c_V \sim (T/T_d - 1)^{-\alpha}, \quad \alpha = 3/5$$

$$l - 1/2 \sim h^{1/\delta}, \quad \delta = 5/2$$

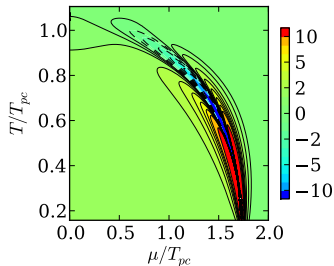
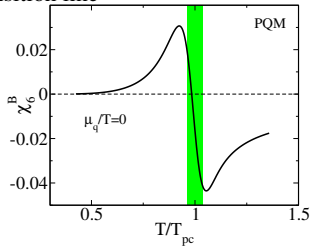
Griffith's scaling relation  $2 - \alpha = \beta(1 + \delta)$  is satisfied.

**GWW PT was also obtained in  $SU(\infty)$  on femtosphere. This suggests that in infinite volume, the Gross-Witten-Wadia transition may be an infrared stable fixed point of a  $SU(\infty)$  gauge theory.**

- $A_0$  modifies not only thermodynamics
- Scattering processes are also modified compared to perturbative/HTL results, e.g.
  - shear viscosity  $\eta \sim l^2$
  - heavy-quark energy loss  $dE/dx \sim l$
  - hard photon production  $q_0 d\Gamma/dq^3 \sim l$  (another step to solve photon  $v_2$  puzzle)
  - dilepton production  $q_0 d\Gamma/dq^3 \sim (1 - l)$
  - work in progress Y. Hidaka, S. Lin, R. Pisarski and V. S.

I had no time to talk about:

- Quark production in Glasma: F. Gelis, K. Kajantie and T. Lappi  
perturbative results not reproduced at large  $k_{\perp}$   
Quark production in Glasma in presence of strong (electro)magnetic field  
work in progress R. Venugopalan and V.S.
- Chiral condensate and its manifestation in HIC (talk by K. Morita):
- signals of criticality on crossover phase transition line
- signals critical point



Kurtosis of baryon number fluctuations

- signals of quarkyonic phase
- background contribution (baryon number conservation, volume fluctuations and etc)

- (Electro)magnetic field in HIC is of hadronic scales. Short lifetime. It gives significant contribution to photon azimuthal anisotropy at RHIC energies. Motivated to study QCD at finite magnetic field.
- Non-trivial classical  $A_0$  field: effective description of Yang-Mills thermodynamics. Modification of scattering processes. In large  $N_c$  limit, deconfinement phase transition in  $SU(\infty)$  theory is of Gross-Witten-Wadia type?! This is suggested both by an effective model and  $SU(\infty)$  on femtosphere.